# Linear Differential Equations Math 102 Section 107 

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\begin{aligned}
& \text { A. } T^{\prime}(t)=E-k T(t) \\
& \text { B. } T^{\prime}(t)=k(E-T(t)) \\
& \text { C. } T^{\prime}(t)=k T(t)-E \\
& \text { D. } T^{\prime}(t)=k(T(t)-E)
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where $E$ (constant) is the surrounding temperature and $k>0$.

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Sanity check: If $E>T$, the temperature increases. If $E<T$, the temperature decreases.
$y^{\prime}=a-b y$

The general solution to the differential equation $y^{\prime}(t)=a-b y(t)$ is

$$
y(t)=\frac{a}{b}+C e^{-b t}
$$

where $C$ can be any constant. The solution satisfying the initial condition $y(0)=0$ is

$$
y(t)=\frac{a}{b}-\frac{a}{b} e^{-b t}=\frac{a}{b}\left(1-e^{-b t}\right)
$$

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Q3. Use the graph to calculate $k_{I V}$.
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C. $k_{I V}=3$
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$k_{I V}$ is the initial rate of drug delivery, and therefore equals the slope of the tangent line at $t=0$.

$$
d(t)=\frac{k_{I V}}{k_{m}}-\frac{k_{T V}}{k_{m}} e^{-k_{m} t}
$$

Q4. How long will it take until the drug levels reach half of the target dose $k_{I V} / k_{m}$ ?
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B. $t=\frac{1}{k_{m} \ln 2}$
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